

A Simple Way for Obtaining the Expression for the Entropy of Fluid

II. The Random Phase Approximation

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Abstract

The more simple technique is used to obtain the analytical expression of the entropy for the square-well fluid in the random phase approximation.

Keywords: Entropy, square-well fluid, random phase approximation

In the previous article (I) was shown a more simple way to obtain the entropy, S , of an equilibrium arbitrary fluid with the hard-core (HC) pair potential,

$$\varphi_{\text{HC}}(r) = \begin{cases} \infty, & r < \sigma \\ \phi(r), & r \geq \sigma \end{cases} . \quad (1)$$

Here, we apply this way to the square-well (SW) model within the random phase approximation (RPA) [1]. The attractive part of $\varphi_{\text{SW}}(r)$ is

$$\phi_{\text{SW}}(r) = \begin{cases} 0, & r < \sigma \\ \varepsilon, & \sigma \leq r < \lambda\sigma \\ 0, & r \geq \lambda\sigma \end{cases} , \quad (2)$$

where ε , λ and σ are the SW parameters.

Then, Fourier transform of Eq. (2) is

$$\phi_{\text{SW}}(q) = \frac{4\pi\varepsilon}{q^3} [\sin(q\lambda\sigma) - \sin(q\sigma) - q\lambda\sigma \cos(q\lambda\sigma) + q\sigma \cos(q\sigma)] . \quad (3)$$

The expression for the Fourier transform of the direct correlation function, $c(r)$, in the SW-RPA approach is

$$c_{\text{SW-RPA}}(q) = c_{\text{HS}}(q) - \beta\phi_{\text{SW}}(q) , \quad (4)$$

where $\beta = (k_{\text{B}}T)^{-1}$, T is the absolute temperature, k_{B} - Boltzmann constant, $c_{\text{HS}}(q)$ - the Fourier transform of the direct correlation function in the hard-sphere (HS) model for which there is the analytical expression [2]. The structure factor, $a(q)$, of the SW system within the RPA is written as

$$a_{\text{SW-RPA}}(q) = \frac{1}{1 - \rho c_{\text{HS}}(q) + \beta\rho\phi_{\text{SW}}(q)} . \quad (5)$$

Finally, using Eq.(10) from (I) we have the following expression for the entropy:

$$S_{\text{SW-RPA}} = S_{\text{HS}} + \frac{k_{\text{B}}}{4\pi^2} \int_0^\infty dq q^2 \left(\beta a_{\text{SW-RPA}}(q) \phi_{\text{SW}}(q) + \frac{1}{\rho} \ln \frac{a_{\text{SW-RPA}}(q)}{a_{\text{HS}}(q)} \right) , \quad (6)$$

where $a_{\text{HS}}(q) = 1/(1 - \rho c_{\text{HS}}(q))$, ρ - mean atomic density.

Since the expression for $a_{\text{SW-RPA}}(q)$ is known, Eq.(6) can be used for actual calculations.

References

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Received: April 30, 2013